

# Models and Analysis of Active Worm Defense

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**Abstract.** The recent proliferation of Internet worms has raised questions about defensive measures. To date most techniques proposed are *passive*, in-so-far as they attempt to block or slow a worm, or detect and filter it. *Active* defenses take the battle to the worm—trying to eliminate or isolate infected hosts, and/or automatically and actively patch susceptible but as-yet-uninfected hosts, without the knowledge of the host’s owner. The concept of active defenses raises important legal and ethical questions that may have inhibited consideration for general use in the Internet. However, active defense may have immediate application when confined to dedicated networks owned by an enterprise or government agency. In this paper we model the behavior and effectiveness of different active worm defenses. Using a discrete stochastic model we prove that these approaches can be strongly ordered in terms of their worm-fighting capability. Using a continuous model we consider effectiveness in terms of the number of hosts that are protected from infection, the total network bandwidth consumed by the worms and the defenses, and the *peak* scanning rate the network endures while the worms and defenses battle. We develop optimality results, and quantitative bounds on defense performance. Our work lays a mathematical foundation for further work in analysis of active worm defense.

## 1 Introduction

A computer worm is so called because it has a life of its own. Once burrowed into a susceptible system, it attempts to propagate through the network. The usual means is through “scans”, it attempts to connect to and infiltrate other hosts throughout the network. Worms interfere with normal use of computers, and exact an economic cost of eradicating them and repairing systems infected by them. Worms have the potential to wreak havoc on the systems they infect, and on the networks they traverse. This potential has been realized already, several times.

The large-scale worm infestations in recent years have triggered several efforts to model worm spread in order to understand how the low-level factors in the propagation mechanism translate into macroscopic behavior, assess threat levels of different worms, and evaluate the effectiveness of detection methods and proposed counter-measures. Staniford appears to have been the first to recognize that the macroscopic propagation of the Code Red v2 worm could be

well modeled through the logistic equation [10]. This model and the equivalent *simple epidemic model* from the epidemic modeling literature (see e.g. [3]) have since been used in several studies [11,6,7,5,13,14]. [12] proposed a model to take removals into account (based on the *general epidemic model*) and [1] proposed a discrete time model.

Our work is unique in considering a wide space of defensive capabilities, and in sample path comparison of them. It is most similar in spirit to [7,1,14] as we use epidemic models to evaluate proposed worm counter-measures. We extend simple epidemic models to consider the interaction of worms and counter-worms and other “active” counter-measures.

For the purpose of illustration the experimental portion of our paper uses parameters reflective of the Code Red v2 worm, released in July 2001. It is important to remember that as far as the mathematics goes, time-scale is irrelevant. Having said that, it is true that very fast worms have had their propagation shaped by the impact they have on the network infrastructure, and the simple mathematical models we develop would not apply.

We focus on worms that spread *autonomously* by probing other systems for vulnerabilities that can be exploited to propagate from one machine to another. This class of worms captures the essence of the rapidly spreading large-scale infestations seen to date, such as Code Red v2, Code Red II, and Nimda in 2001, and Slammer, Blaster, and Welchia in 2003. Thus, we deliberately exclude most typical email born viruses that require a user action to enable infection. In contrast, worms such as Slammer have proven that the time-scales involved for fast moving autonomously propagating worms can be so short that human intervention to stop them is impossible. Consequently, this class of worms poses a substantial threat and a trigger for development of automated defensive mechanisms, such as those we consider in this paper.

In the wake of one worm attack (Blaster), a counter-worm (Welchia) was launched that sought hosts infected by Blaster, attempted to patch them, and use them to find other infected hosts. Whatever the intentions of the authors might have been, Welchia had consequences as bad or worse than Blaster—it was harder to get rid of, and effectively created a denial-of-service attack on patch servers, so that people trying manually to protect their systems had a harder time doing so. The question is raised therefore of the effectiveness and impact that an “active defense” might have. We examine this question agnostically and without overt consideration of the legal and ethical issues raised by wide-spread active defense. It is enough for us that an organization as large as the United States Department of Defense could mandate such measures on its own gargantuan networks; we seek to understand the power and the limitations of active defense deployment, should they be deployed. Our approach is analytic. We consider four aspects of active defense—patching uninfected hosts, increasing the active defense population by using uninfected hosts that are susceptible to the worm, suppression of infected hosts discovered through scans, and suppression of infected hosts discovered through scans *and* traffic analysis. Using a very general discrete stochastic model, we show that adding each capability (in that order)

to the active defense assumptions results in a stochastically stronger increase in worm-fighting power. Using a continuous model we quantify some aspects of active defense behavior, and prove some results about it.

## 2 Active Defense

Imagine a network where there are  $N$  hosts with a particular set of vulnerabilities, and then a worm is released that is able to exploit one or more of these. We suppose that a host infected by this worm scans the network looking for vulnerable hosts it may infect. We assume that a scan consists of a random selection of an IP address— if that host is susceptible and uninfected it immediately becomes infected. In our discrete model we assume that the address selection is oblivious to the state of the network. This means that non-uniform random scanning can be accommodated in the model, so long as the sampling is not affected by any knowledge of other hosts, infected or not. This does not preclude the sort of stratified sampling seen in some worms (where hosts “closer” to the infected one are sampled with higher probability), but it does preclude a dynamic partitioning of the search space based on coordination among infected hosts. We assume a random delay of time between successive scans from a host, once again assuming that the sampling is independent of network state.

Under these assumptions we can picture the behavior of a worm on a timeline populated with scan events. Each scan event has a source and destination identity. Each of the susceptible hosts has a state of uninfected, or infected. A scan event that has an uninfected host as destination changes that host’s state, and thereafter it contributes to the scanning. (It is straightforward to augment the model to account for latency between when a scan is sent and when it is received, we have not done so for simplicity of exposition).

### 2.1 Defense Capabilities

At time 0 the worm is launched from  $w_0$  of the  $N$  susceptible hosts. Each infected host scans the network using a randomized strategy that is oblivious to the network state. We assume that the worm immediately inhibits further penetration through the same vulnerability, but that a counter-worm scanning it can recognize the presence of the worm, (e.g. through observation of banner information that the host’s software returns, revealing a version and build that admits penetration through the known vulnerability).

We envision a model of *active defense* as follows. At time  $T_0 > 0$ , some  $I_0$  hosts begin executing an active defense. Each of those hosts scans, using a strategy (probably, but not necessarily random) that is oblivious to the network state. Whenever one of these scans targets a susceptible but uninfected host, that host becomes (instantly!) patched to prevent infection from the worm. We call this a **simple patch** defense. This defense (and all the others we consider) presumes that the defensive mechanism was prepared before the worm was launched. So-called 0-day attacks, ones that exploit previously unknown exploits, are fairly

rare. The vulnerabilities that worms exploit are more typically announced when discovered, often with patches available. More often than not the patch code reveals details worm writers use to target as-yet unpatched systems. It is not unreasonable to suppose then that patching defense code could be crafted along with the patch. A reason for not releasing the patching defense in anticipation of a worm is that the release would contain the code to exploit the vulnerability, with no work or further cleverness needed by a worm-writer. A patching defense must be coupled with a worm-detection mechanism, such as those proposed in [5,13].

One could increase the presence of the active defense by increasing the number of hosts running the patching logic. So we define a **spreading patch** defense as one where, when an uninfected susceptible host is scanned, it is endowed with a counter-worm that both patches, and scans. While the number of patching hosts remains constant in a simple patching defense, it grows in a spreading patch defense. Such a mechanism has been seen in the wild [4].

A third presumed defensive capability is worm suppression. Suppose that when a patching host scans an infected host it is able to identify the host as infected, and to suppress the infecting scans from being seen elsewhere, thereafter—it is able to *nullify* the infected host. For example, the spreading-patch worm might have an ability to cause the infection traffic to be filtered by a nearby router; another way might be if every machine in an organization had a “lock”, such that when the proper “key” is applied, some or all of that machine’s external communication is inhibited—an organization’s active defense posture would include selective suppression of machines thought to be infected. For our purposes, the important thing is that the infected host be discovered by a scan, and that thereafter it is no longer a source of infection. We call this a **nullifying** defense.

A fourth presumed defensive capability takes advantage of the fact that some attacks are complex enough to require that the attacking host use its legitimate IP address as source in its packets (and we may anticipate that in the future the ability to spoof source addresses will become much diminished, through more active router verification procedures). Because of this, a patching host that *receives* a scan from an infected host could turn around and nullify the infection. In this **sniper** defense one expects that infected hosts diminish in number faster than when they are discovered merely by scans.

## 2.2 Metrics

There are different ways of assessing an active defense. When host integrity is paramount, then an appropriate metric is the number of hosts infected by the worm. We define  $I(D, t)$  to be the cumulative number of hosts infected by time  $t$  under defense  $D$ . This metric is a random variable; we will say that  $D_i$  is more *powerful* than  $D_j$  if for all  $t > 0$  and  $n > 0$ ,

$$\Pr\{I(D_i, t) > n\} \leq \Pr\{I(D_j, t) > n\}.$$

When this relationship holds we say that the distribution (with respect to randomness due to sampling) of  $I(D_j, t)$  is *stochastically larger* than  $I(D_i, t)$ [9],

denoted  $I(D_j, t) \geq_{st} I(D_i, t)$ .  $D_i$  is more powerful in the sense that it does a better job at preventing susceptible hosts from becoming infected. This stochastic ordering is strong in its implications. It is known that if  $X \geq_{st} Y$  and  $f$  is any increasing function, then  $E[f(X)] \geq E[f(Y)]$ . This has bearing then for any system metric that depends monotonically on infection counts, e.g., the probability of system failure would likely be monotone increasing in the number of infected hosts.

An active defense may increase the overall scanning activity on the network, and there is evidence that intense scanning can harm the network [2]. When network health is the principle concern, then measures of scanning history, and/or scanning intensity are appropriate. If  $\lambda(D, t)$  denotes the scanning rate due to both worm and defense  $D$ , then we assess a defense in terms of its peak scanning rates over some interval  $[0, t]$ :

$$\max_{0 < s < t} \{\lambda(D, s)\}$$

We might also assess it through its aggregate scanning rates (the space-time product) over some interval  $[0, t]$ :

$$\int_0^t \lambda(D, s) ds.$$

### 3 Ordering of Defenses

Intuition suggests that the four active defenses (five, if we include the empty defense) we've outlined might be ordered in terms of power. We now show that this is exactly the case. In the comparisons made, we use the *Common Sample Path* assumption, that once a host is infected (or takes on the counter-worm), its scanning behavior is completely determined by a random number stream that is independent of any other. When we compare two defenses, we assume that a host uses that same stream in both systems, which allows us to compare the two systems on commonly constructed sample paths. The implication is that once a host is infected (or starts to run a counter-worm), its sequence of inter-scan delays are the same in both systems, and the pattern of hosts scanned are the same in both systems. Thus, if the two systems cause a host to be infected at the same instant, on the sample paths being compared that host will scan exactly the hosts at exactly the same time, in both systems.

The results to follow are based on a construction we call the Sample Path Graph (SPG). For every susceptible host  $h_i$  let  $\mathcal{I}_i$  be a sequence of pairs  $(t_i, dst_i)$  identifying the time since the host started infection scanning, and a destination  $dst_i$  of a scan.  $\mathcal{I}_i$  is ordered by increasing values of  $t_i$ . We define  $\mathcal{C}_i$  similarly, describing the scanning pattern once a host starts running a counter-worm. We construct a graph whose nodes represent hosts that are assumed to be infected already at time 0 (and which have scanning sequences), nodes representing hosts that eventually start counter-worm scans (with their own scanning sequences), and susceptible hosts. The graph contains a directed edge for every potential

scan described in the sets  $\{\mathcal{I}_i\}$  and  $\{\mathcal{C}_i\}$  whose target was susceptible at time 0. The edge is directed from the source of the scan to the target; an edge will be called an *infection edge* or *countering edge*, depending on whether it comes from an infection or counter-worm sequence, respectively. The node for host  $h_i$  will have values  $S(h_i)$  recording the earliest time it was scanned by an infected host, and  $C(h_i)$  recording the earliest time it was scanned by a host running a counter-worm. Some of the edges are labeled with the time of the scan—these edges are particularly important in our analysis. The values of  $S(h_i)$  and  $C(h_i)$ , the edges labeled and the values of those labels all depend on the particular defense. However, common to those defenses are the following rules:

- All hosts assumed to be already infected at time 0 label each of their edges with the corresponding scan time;
- all hosts that are used to start the counter-worm label each of their edges with  $T_0$  plus the corresponding scan time offset contained in the scan sequence.

The differences between different defense's SPGs are characterized as follows:

### Empty Defense ( $D_0$ )

1. The node for host  $h_i$  defines  $S(h_i)$  to be the smallest label among all labeled infection edges directed to it;  $S(h_i) = \infty$  if no such edge exists.
2. A host  $h_i$  labels the infection edge corresponding to the  $j^{\text{th}}$  element of  $\mathcal{I}_i$  (say,  $(s_j, dst_j)$ ) with value  $S(h_i) + s_j$ ,  $j = 1, 2, \dots$ .

The difference between the simple patch defense and the empty defense is that susceptible hosts are protected from infection if they are touched by a countering scan before being touched by an infection scan.

### Simple Patch ( $D_1$ )

1. Item (1) from the Empty Defense rules.
2. The node for host  $h_i$  defines  $C(h_i)$  to be the smallest label among all labeled countering edges directed to it;  $C(h_i) = \infty$  if no such edge exists.
3. If  $S(h_i) < C(h_i)$  the node labels the infection edge corresponding to the  $j^{\text{th}}$  element of  $\mathcal{I}_i$  (say,  $(s_j, dst_j)$ ) with value  $S(h_i) + s_j$ ,  $j = 1, 2, \dots$ .
4. If  $C(h_i) < S(h_i)$  the node does not label any of its edges.

The difference between a spreading patch defense and a simple patch one is that a host that receives a countering scan before any infection scan becomes host to counter-worm software, and generates its own countering scans.

### Spreading Patch ( $D_2$ )

1. Items (1) from the Empty Defense rules, (2), and (3) from the Simple Patch rules.
2. If  $C(h_i) < S(h_i)$  the node labels the countering edge corresponding to the  $j^{\text{th}}$  element of  $\mathcal{C}_i$  (say,  $(s_j, dst_j)$ ) with value  $C(h_i) + s_j$ ,  $j = 1, 2, \dots$ .

The difference between a nullifying defense and a spreading patch defense is that when a countering scan reaches a host that is already sending infection scans, the infection scans stop.

### Nullifying Defense ( $D_3$ )

1. Item (1) from the Empty Defense rules, item (2) from the Simple Patch rules, and item (2) from the spreading patch rules.
2. If  $S(h_i) < C(h_i)$  the node labels the infection edge corresponding to the  $j^{\text{th}}$  element of  $\mathcal{C}_i$  (say,  $(s_j, dst_j)$ ) with value  $C(h_i) + s_j$ , for all  $j$  such that  $S(h_i) + s_j \leq C(h_i)$ .

And finally, the difference between a sniper defense and a nullifying defense is that infection scans that encounter hosts running countering scans cause the host sending the infection scan to cease. This may occur before the host is itself scanned by a countering scan (which has the same nullifying effect).

### Sniper Defense ( $D_4$ )

1. Item (1) from Empty Defense rules, item (2) from the Simple Patch rules, item (2) from the Spreading Patch rules.
2. If  $S(h_i) < C(h_i)$ , let  $k$  be the smallest index for  $(s_k, dst_k) \in \mathcal{I}_i$  such that  $S(h_i) + s_k > C(dst_k)$ , and define  $K_i = S(h_i) + s_k$ . The node for  $h_i$  labels the infection edge corresponding to the  $j^{\text{th}}$  element of  $\mathcal{C}_i$  (say,  $(s_j, dst_j)$ ) with value  $C(h_i) + s_j$ , for all  $j$  such that  $S(h_i) + s_j \leq \min\{C(h_i), K_i\}$ .

The construction above make the conditions under which a given infection edge is labeled increasingly restrictive, as we move through sequence of defenses. This implies that if we choose a host  $h_i$  and defenses  $D_a$  and  $D_b$  with  $a < b$ , then the set of labeled incoming infection edges it has in the SPG for  $D_b$  is a subset of the labeled incoming infection edges it has in the SPG for  $D_a$ . This fact enables us to prove the central results comparing different defenses.

**Lemma 1.** *Consider two defenses  $D_a$  and  $D_b$ ,  $a < b$ , under identical boundary conditions. Let  $G_a$  and  $G_b$  be corresponding Sample Path Graphs constructed under the Common Sample Path assumption, and let  $S^{(y)}(h)$  and  $C^{(y)}(h)$  denote the  $S(h)$  and  $C(h)$  variables for host  $h$  under defense  $y \in \{a, b\}$ . Then for every host  $h$ ,  $S^{(a)}(h) \leq S^{(b)}(h)$  and  $C^{(b)}(h) \leq C^{(a)}(h)$ .*

*Proof.* Without loss of generality renumber the hosts by increasing value of  $S^{(b)}(h)$ , we induct on this order. Consider the base case of  $h_0$ . Both  $S^{(a)}(h_0)$  and  $S^{(b)}(h_0)$  are defined by edges from hosts assumed to be infected at time 0, and are thus identical. In both  $G_a$  and  $G_b$  host  $h_0$  gets the same set of labeled countering edges from the initial set of hosts running the defense, and  $C(h_0)$  in both graphs is no larger than the smallest of these labels. However, in  $G_b$  there may be more countering edges labeled, and hence the possibility of a shorter path to  $h_0$  through those edges, whence  $C^{(b)}(h_0) \leq C^{(a)}(h_0)$  and the induction base is established. For the induction hypothesis we assume that the assertion

is true for all hosts  $h_0, h_1, \dots, h_{n-1}$  for some  $n$ , and consider host  $h_n$ . Let  $e$  be the labeled infection edge coming into  $h_n$  whose label defines  $S^{(b)}(h_n)$ , and consider its manifestation  $e'$  in  $G_a$ . By the construction of SPG's, an infection edge may appear labeled in the SPG of one defense  $D_u$  and not another  $D_v$  if its target  $h_y$  has a smaller value  $C(h_y)$  in  $G_v$  than in  $G_u$ , or if  $G_v$  is nullifying and scans a countering host. In all cases the only way a labeled edge appears in  $G_u$  and not  $G_v$  is when  $u < v$ . Consequently  $e'$  appears labeled in  $G_a$ . This in turn implies that the node  $h_m$  from which  $e'$  is directed satisfies  $m < n$ , as it is directed from the same node in both  $G_a$  and  $G_b$ . By the induction hypothesis  $S^{(a)}(h_m) \leq S^{(b)}(h_m)$ , which implies that the label on  $e'$  is no larger than the label on  $e$ , and thus, that  $S^{(a)}(h_n) \leq S^{(b)}(h_n)$ . A similar argument shows that the labeled countering edge  $g$  which defines  $C^{(a)}(h_n)$  (when this exists) has a labeled counter-part  $g'$  in  $C^{(b)}(h_n)$ , whose label is no larger in  $G_b$  than it is in  $G_a$ , and thus that  $C^{(b)}(h_n) \leq C^{(a)}(h_n)$ . This completes the induction.  $\square$

From this result comes the main result.

**Theorem 1.** *For defense  $D_i$  and every time  $t$ , let  $I(D_i, t)$  denote the number of hosts infected by time  $t$  (including those that later become nullified). Then for  $a < b$ ,  $I(D_a, t) \geq_{st} I(D_b, t)$  for every  $t \geq 0$ .*

*Proof.* Lemma 1 shows that for any sample path of scans and every time  $t$ , the number of hosts  $h$  with  $S^{(a)}(h) \leq t$  is greater than or equal to the number of hosts  $h$  with  $S^{(b)}(h) \leq t$ . For any sample path these counts define the random variables  $I(D_a, t)$  and  $I(D_b, t)$ . Coupling results in [9] establish the result.  $\square$

These results show that the difference between defenses is structural, and strong. The results are very general, free of distributional assumptions other than independent of sampling from network state. However, they don't give much insight into how well these defenses perform.

There is one exception, in the special case where the counter-worm has the same scanning characteristics as the worm. Then we may assume that whenever a host is entered either by a worm, or a counter-worm, its pattern of scans (inter-scan delays, sequence of targets scanned) is the same under any defense. From the point of view of the same path analysis we've done, it means that whenever a node is triggered to scan we may assume it does so with exactly the same pattern regardless of if that is an infection or countering scan. This means that any host that scans in an empty defense also does so in a spreading patch defense, only possibly earlier (if the scan is a countering scan).

These observations establish the theorem.

**Theorem 2.** *Suppose that the scanning structure of the counter-worm is identical to the worm. For every time  $t$  let  $\lambda(D_0, t)$  and  $\lambda(D_2, t)$  denote the instantaneous number of hosts scanning under the empty defense and spreading patch defense, respectively. Then for every  $t$ ,  $\lambda(D_2, t) \geq_{st} \lambda(D_0, t)$ .*

This theorem is a strong statement about a condition when adding defense is worse, from the point of view of the network. Increasing functions of  $\lambda(D, t)$  include the peak number of hosts scanning over an interval, the space-time product

of the bandwidth devoted to scanning, the probability of network partition, and so on. The stochastic ordering asserts that the expectation of each of these is larger when we use a spreading patch defense than when we use no defense at all.

## 4 Epidemic Models

We use a style of modeling based on well known models from the epidemic modeling literature. In typical simple epidemic models we consider a fixed population of  $N$ , where each individual is susceptible to infection, and each individual will, at any given time, be in one of a small set of predefined states. For instance, in the *simple epidemic model* [3] (aka the *SI* model and equivalent to the logistic equation) an individual is either in state  $S$  (susceptible to infection) or  $I$  (infected). We denote by  $s(t)$  and  $i(t)$  the number of individuals in state  $S$  and  $I$  respectively at time  $t$ , and thus  $\forall t, s(t) + i(t) = N$ . For large enough populations, the mean rate of state changes  $S \rightarrow I$  can be modeled as:

$$\begin{aligned}\frac{ds(t)}{dt} &= -\beta s(t)i(t) \\ \frac{di(t)}{dt} &= \beta s(t)i(t)\end{aligned}$$

where the constant  $\beta$  is the *infection parameter*, i.e. the pairwise rate of infection.  $\beta$  reflects the aggregate scanning rate of an infected host, as well as the mean probability of selecting a given address for an individual probe attempt. The system boundary conditions are given by the number of initially susceptible hosts  $s(0)$  and initially infected hosts  $i(0)$ . This model rests on assumptions of *homogeneous mixing*, which correspond well to a uniformly random scanning worm spreading freely through a network, so in the following we will refer to this the **Random Scanning Worm Model**.

Other scanning strategies are possible. For instance, worms such as Code Red II, Nimda, Blaster, and Welchia utilized preferential (rather than uniform) scanning techniques where addresses close in the address space to the scanning host's would be probed with higher probability. Other suggested possibilities include a "Divide-and-Conquer" approach to probing the address space (see "partitioned permutation scan" in [11]). Here each worm is assigned a disjoint fraction of the address space to probe.

Other simple tricks for speeding up the propagation have been suggested, such as the use of pre-compiled hit-lists or using inter-domain routing tables to only scan routed space [14]. We can incorporate these into our framework; hit-listed hosts can be made to be infected as a boundary condition, and use of routing tables just increases  $\beta$  to reflect that the scanning is over a smaller address space.

The early stage of infection is the most critical time for any counter-measures to be effective. Since the worms behave similarly in the early stages we will, in the following, focus on random scanning worms as this is the type of worm that has been observed in the wild to date.

In [7], Moore *et al.* note that when considering the effectiveness of defensive measures, it is preferable to consider the quantiles of infection rather than the mean number of infections due to the variability inherent in the early stages of infection growth. However, we prefer to use these mean-value based models, because they lend themselves to analysis in a way that stochastic simulations do not. Moreover, we are mainly concerned with the relative performance of different defenses as we compare them, and we believe that the relative performance can be credibly determined in terms of the mean, even though the predicted mean absolute performance should be viewed with caution.

The simple epidemic model we study is suitable only in contexts where the worm scanning is unaffected by the network topology. This assumption is fine for worms whose mass and scan rates aren't constrained by bandwidth (as was the case with Code Red, and others), but is not acceptable when network constraints hinder worm growth. In related work we are exploring how to incorporate network constraints into efficient simulation of worm dynamics [8].

#### 4.1 Spreading Patch Counter-Worm

Consider the spreading patch counter-worm model discussed earlier, and assume that it uses the same vulnerability and propagation strategy as the original worm. Under these assumptions the second worm will spread at (approximately) the same rate as the original worm, seeking the same susceptible population of hosts. A simple model is:

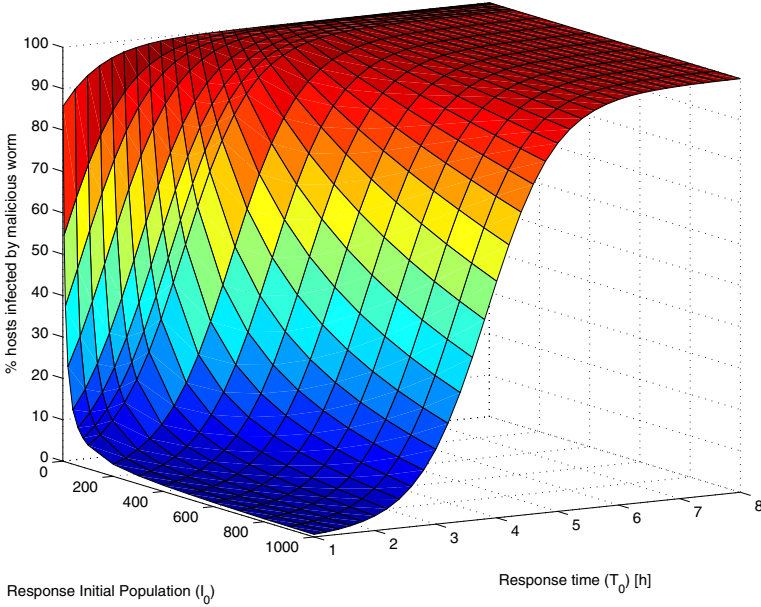
$$\begin{aligned}\frac{ds(t)}{dt} &= -\beta s(t)(i_b(t) + i_g(t)) \\ \frac{di_b(t)}{dt} &= \beta s(t)i_b(t) \\ \frac{di_g(t)}{dt} &= \beta s(t)i_g(t)\end{aligned}$$

where  $i_b$  refers to infections by the malicious (bad) worm and  $i_g$  refers to infections by the spreading-patch (good) worm. Given  $\beta$  and  $i_b(0)$ , system behavior is governed by the time  $T_0$  at which spreading-patch worms are released, and the number of worms  $I_0$  released then. We assume that the spreading-patch worms are launched on “friendly” machines that are not part of the susceptible or infected set.

Spreading-patch worm effectiveness as a function of response time and initial population is shown in Figure 1. An effective response requires a combination of low response time and a sufficiently large initial population. Launching a single counter-worm has little effect, and the window of opportunity for launching even a thousand spreading-patch worms disappears after a couple of hours.

At  $T_0$ ,  $i_b(T_0)$  hosts have succumbed to the original worm and there are  $s(T_0)$  remaining susceptibles. How many spreading-patch worms must be launched to protect a given fraction  $p$  of those remaining susceptibles? If we consider the fraction of infection growth due to the spreading-patch worm

$$\frac{di_g(t)/dt}{di_g(t)/dt + di_b(t)/dt} = \frac{i_g(t)}{i_g(t) + i_b(t)}$$



**Fig. 1.** Effectiveness of spreading-patch worm as a function of response time and initial counter-worm population

we see that since the propagation rates are the same, the proportions of the susceptible population consumed by each worm from  $T_0$  onwards simply correspond to their proportion of the population at  $T_0$ . Thus, ultimately the fraction of hosts which were susceptible at  $T_0$ , but eventually are patched is

$$p = \frac{I_0}{I_0 + i_b(T_0)}.$$

Solving for  $I_0$  we get

$$I_0 = \left( \frac{p}{1-p} \right) \cdot i_b(T_0) \quad (1)$$

Thus, the fraction of all susceptibles  $s(0)$  that will be protected is

$$\tilde{p} = \frac{p \cdot s(T_0)}{s(0)} = \frac{p[s(0) - i_b(T_0)]}{s(0)} = p \left( 1 - \frac{i_b(T_0)}{s(0)} \right)$$

If the infection is caught early on, then  $i_b(T_0) \ll s(0)$ , and the protected fraction  $\tilde{p} \approx p$ . Thus, equation (1) can be used as a guideline for selecting  $I_0$  given only an estimate of how many hosts have been infected at the time of response ( $i_b(T_0)$ ), assuming that the response occurs early. Such an estimate can reasonably be obtained by analysis of observed scanning behavior.

The spreading-patch worm model considered here assumes only that it scans at the same rate as the original worm. It does not assume any information about the malicious worm and its behavior. As worms to date have exploited vulnerabilities that were previously known, it is not unreasonable to suppose that a patching worm might be developed when the vulnerability is identified (but before it is announced), against the possibility of needing to use it. Such a worm would not be launched before needed, because it could be captured and analyzed for the means to exploit the vulnerability. However, the fact that the spreading-patch worm has higher impact on the network (Theorem 2) than no defense at all encourages us to explore counter-worms that have stronger capabilities in worm identification and suppression, with smaller impact on the network.

## 4.2 Nullifying Defense

Next we develop a continuous model of the nullifying defense. Using notation similar to that for the spreading patch defense, we develop state equations

$$\begin{aligned}\frac{ds(t)}{dt} &= -\beta s(t)(i_b(t) + i_g(t)) \\ \frac{di_b(t)}{dt} &= \beta s(t)i_b(t) - \beta i_b(t)i_g(t) \\ \frac{di_g(t)}{dt} &= \beta s(t)i_g(t)\end{aligned}$$

Here we see a new component to  $(di_b(t)/dt)$ , the subtraction of hosts due to being scanned by the counter-worm.

Under our assumptions, in the limit of increasing time  $t$ , the aggregate scan rate under the spreading patch defense is proportional to the number of “outside” spreading-patch hosts  $I_0$  plus the initial susceptible population size  $s(0)$ —eventually every susceptible host is running either the worm, or the counter-worm. However, in the case of nullifying worms, the aggregate *peak* scan rate may be smaller than the aggregate peak scan rate of the unfettered worm.

**Theorem 3.** *Suppose that  $I_0$  initial nullifying worms are released at time  $T_0$ . If  $I_0 \leq i_b(T_0)$ , then the aggregate peak scan rate using the nullifying worm is less than the peak scan rate of the unfettered worm.*

*Proof.* Let  $i_n(t)$  be the aggregate number of infected hosts that a nullifying defense has identified and contained by time  $t$ , and let  $e(t)$  be the number of formerly susceptible hosts that have been “enlisted” to run the nullifying worm. At any time  $t$  the aggregate scan rate of a defense is proportional to  $i_b(t) + i_g(t) = i_b(t) + I_0 + e(t)$ . From the invariant  $s(0) = s(t) + i_b(t) + i_n(t) + e(t)$  we replace  $e(t)$  in the scan rate expression to see that the scan rate at  $t$  is proportional to  $I_0 + s(0) - s(t) - i_n(t)$ . The maximum value of this term will always be less than  $s(0)$  if  $I_0 < s(t) + i_n(t)$  for all  $t$ . Examination of derivatives shows that  $s(t) + i_n(t)$  is monotone decreasing, hence its lowest value is the asymptotic value of  $i_n(t)$ ,

say,  $\mathcal{N} = \lim_{t \rightarrow \infty} i_n(t)$ . By assumption  $I_0 \leq i_b(T_0)$ , and clearly  $i_b(T_0) < \mathcal{N}$ . The conclusion follows immediately.  $\square$

It is interesting to compare this result—which says if one *limits* the initial infection of the counter-worm you can bound the peak scan rate from above, with the spreading-patch defense results which turn these inequalities around. With the spreading-patch defense a minimum size of the release needs to be  $I_0 > i_b(T_0)$  to give it enough mass to overtake the original worm. But because the nullifying worm fights by decreasing the number of scanning worms, it gets by with a smaller initial counter-worm population.

Another capability a nullifying defense could have is that it stop all defensive scanning, upon centralized command. This would help mitigate against overwhelming the network with scans from the defenses (a characteristic reported of the counter-worms seen in the wild). Denote the defensive worm stopping time by  $t_s$ . The modified state equations after time  $t_s$  are

$$\frac{ds(t)}{dt} = -\beta s(t)i_b(t) \quad (2)$$

$$\frac{di_b(t)}{dt} = \beta s(t)i_b(t) \quad (3)$$

$$\frac{di_g(t)}{dt} = 0 \quad (4)$$

Figure 2 illustrates the evolution of system state where the nullifying defense is propagating without stopping. Also shown, is the resulting peak total population (directly related to peak bandwidth in our model) as a function of stopping time  $t_s$ . Taking the time at which the defensive worms are stopped as a control parameter, we see that the minimized peak scan rate obtained by optimally selecting the stopping time is no larger than the peak scan rate if the defenses are never turned off. This capability can only improve the peak scan rate over that of the earlier nullifying defense we considered.

For  $t < t_s$  the scan rate is proportional to  $i_b(t) + i_g(t)$ ; the peak scan rate achieved after  $t_s$  is proportional to  $i_b(t_s) + s(t_s)$ , for the original worm will eventually infect all hosts left unprotected once we stop the defensive scans. Examination of derivatives shows that

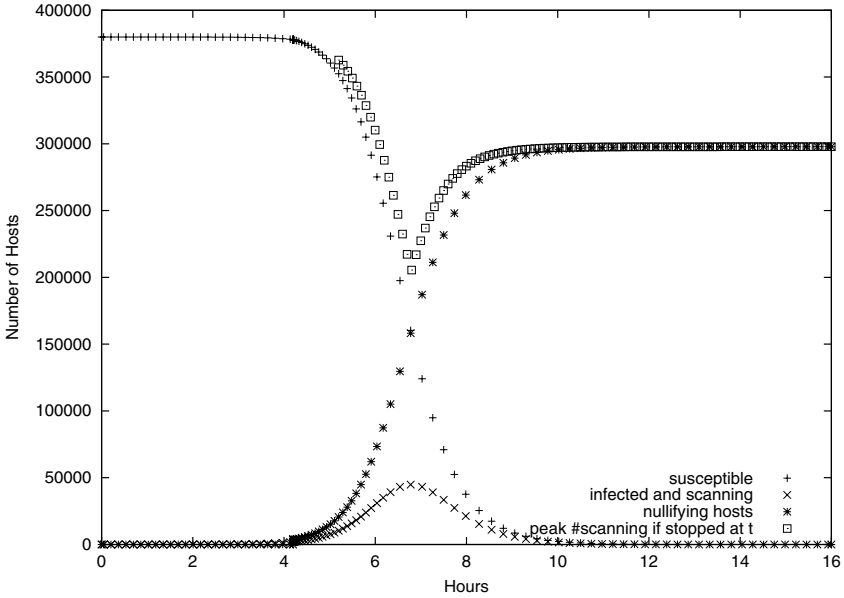
$$\frac{d(i_b(t) + i_g(t))}{dt} = \beta(i_b(t)(s(t) - i_g(t)) + s(t)i_g(t))$$

which we observe is positive at least as long as  $s(t) \geq i_g(t)$ . Likewise, derivatives show that  $i_b(t) + s(t)$  is a decreasing function :

$$\frac{d(i_b(t) + s(t))}{dt} = -\beta i_g(t)(i_b(t) + s(t)).$$

If the nullifying defense scans are stopped at  $t_s$  with  $s(t_s) \geq i_g(t_s)$  we are assured that the peak scanning rate of the system is

$$\max\{i_b(t_s) + i_g(t_s), i_b(t_s) + s(t_s)\}.$$



**Fig. 2.** Peak bandwidth used by the nullifying defense (and original worm) as a function of when it is switched off

So long as the first argument is increasing and the second argument is decreasing, the stopping time that minimizes the maximum occurs when the arguments are equal, e.g., when  $i_g(t) = s(t)$ ; since  $i_b(t) + i_g(t)$  is still monotone at this point,  $t_s$  minimizing the peak aggregate scanning rate satisfies  $i_g(t_s) = s(t_s)$ .

We are in a position now to quantify the performance of a defensive worm. We can show that the minimal peak number of hosts scanning is at least  $(1/3)(s(0) + I_0)$ , provided that  $I_0 \geq i_b(T_0)$ , a result which we state formally.

**Theorem 4.** *Consider a nullifying defense that is launched at time  $T_0$  with  $I_0 \geq i_b(T_0)$  initial instances, and whose scans can be stopped on command. The stopping time  $t_s$  which minimizes peak scanning is the unique solution to  $i_g(t_s) = s(t_s)$ . A lower bound on the peak number of hosts scanning is  $(1/3)(s(0) + I_0)$ .*

*Proof.* We first note that under the assumption  $I_0 = i_g(T_0) > i_b(T_0)$ , that  $i_g(t) \geq i_b(t)$  for all  $t \geq T_0$ . This is a result of both the worm and the counter-worm competing for exactly the same pool of susceptible hosts—at the same rate (per host)—with the counter-worm starting with at least as many hosts as are in the infection at the time the counter-worm is released. A consequence is that the time  $t_s$  when  $s(t_s) = i_g(t_s)$  occurs before the time  $t_b$  that  $s(t_b) = i_b(t_b)$ . This fact turns out to be important as we ask for conditions under which  $i_g(t) \geq i_n(t)$ , where  $i_n(t)$  is the number of infected hosts that have been nullified. We know that  $i_g(T_0) > i_n(T_0)$ ; analysis of the derivative of  $i_g(t) - i_n(t)$  shows that this difference grows so long as  $s(t) \geq i_b(t)$ —a condition which can only occur after the stopping time  $t_s$ . Finally, we note the invariant

$$i_b(t) + i_g(t) + i_n(t) + s(t) = s(0) + I_0.$$

At the stopping time,  $s(t_s) = i_g(t_s)$ , and  $i_g(t_s) > i_n(t_s)$ , whence

$$i_b(t_s) + 3i_g(t_s) > s(0) + I_0.$$

It follows that  $i_b(t_s) + i_g(t_s) > (1/3)(s(0) + I_0)$ . □

We see that under the theorem’s assumptions, the capabilities nullifying defenses have over spreading-patch defenses (suppress an infected host’s scans, stop the “good worm” scanning) serve to give it greater power, but the peak number of hosts scanning (both worm and counter-worm) is still at least one third of the initial susceptible population. It should be noted that this result depends significantly on an assumption that the counter-worm’s scan rate is identical to the worm’s. We are exploring the consequences of relaxing this assumption, as well as pushing on looking for ways of countering worms with increasing power, while reducing the impact on the network.

## 5 Conclusions

This paper studies active defenses against Internet worms. We use discrete and continuous mathematical models to study a hierarchy of worm fighting capabilities. We are able to prove a number of results about these models, including

- strong stochastic ordering of infection counts in a hierarchy of five defense types;
- that a simple counter-worm defense has a stochastically larger aggregate scanning intensity than does the unfettered worm;
- that by starting a defense with enough outside hosts scanning to implant counter-worms, any desired fraction of the remaining susceptible hosts can be protected from a worm;
- that by starting a nullifying defense with few enough outside hosts, the peak scanning intensity is less than the unfettered worm;
- even when peak scanning time is minimized under the nullifying defense, it is still the case that the peak number of hosts scanning is at least 1/3 of the total number of susceptibles;

There is much work yet to be done. This paper does not address the very significant problem of quickly and automatically *detecting* when a worm attack has been launched—we have looked only at the relative effectiveness of measures put into place after the detection. Our experiments of effectiveness of defense *as a function of response time* (Figure 1) show that rapid detection is absolutely critical.

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